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Science as (Historical) Narrative

M. Norton Wise

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Abstract The traditional mode of explanation in physics via deduction from partial differential equations is contrasted here with explanation via simulations. I argue that the different technologies employed constitute different languages, which support different sorts of narratives. The narratives that accompany simulations and articulate their meaning are typically historical or natural historical in kind. They explain complex phenomena by *growing* them rather than by referring them to general laws. Examples of such growth simulations and growth narratives come from the evolution of wave functions in quantum chaos, snowflake formation, and Etruscan genetics. The examples suggest a few concluding remarks on historical explanation.

1 Introduction

The following reflections derive from two quite general features of contemporary science. The first concerns the all-important role of technologies in science. The second is a remarkable change in modes of scientific explanation. I will be seeking a relation between these two features through the role of narrative.

Concerning technologies, many historians have come to regard technologies in science as providing tools to think with, as well as to work with materially. We regularly analyze their role as active agents of thought and action, or as technologies of knowledge. But in this the history of science mirrors another major development. The sciences themselves have become much more intimately tied up with

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M. N. Wise (✉)
Department of History and Center for Society and Genetics, University of California, Los Angeles (UCLA), Los Angeles, CA 90095-1473, USA
e-mail: nortonw@history.ucla.edu

technologies than ever before and scientists are changing their view of their role. A quite specific statement comes from Shirley Tilghman, molecular biologist and president of Princeton University, asserting that "while progress in science and engineering can be catalyzed by entirely new theoretical insights, it is more often the case that revolutionary advances spring from the arrival on the scene of new technologies that allow investigators to explore unresolved questions with new tools, or to ask previously unapproachable questions" (Tilghman 2009). More broadly, consider the Nobel Prizes in physics awarded in 2007 for the hard disk (giant magnetoresistance) and again in October 2009 for fiber optics and digital cameras (charge coupled devices). Not everyone welcomes the change. Exemplary is a recent article by Paul Forman decrying the primacy of technology in postmodernity (Forman 2007).

Turning to modes of explanation, a number of historians of science have observed that explanations are becoming historicized in surprising ways. Sylvan Schweber pioneered such reflections and others have developed similar views (Schweber 1993; Dalmédico 2004, pp. 83–85; Wise 2004; Creager et al. 2007). With respect to physics in particular, the ideal of deduction from timeless universal laws has been losing its grip in favor of forms of explanation that have an increasingly historical character.

These two observations—on technology and on historicity—suggest that we need a way to talk about explanations that incorporates the relationship between them. One possibility, and the one I will propose here, is to treat technologies of knowledge as languages that support particular forms of explanatory narratives.[1] I will develop this viewpoint with respect to mathematical physics, comparing explanation via deduction from partial differential equations with explanation via simulations, with extension to an historical simulation. In conclusion, I will suggest that traditional philosophy of history requires a major updating to compare narrative history, not with deduction from general laws (a comparison that has been ubiquitous in the field for 50 years) but with the more narrative form of simulations.

2 Written Language as Material Technology

A starting point can be had from writings in literary criticism that explicitly treat written language as technology. A recent example is Russell Berman's *Fiction Sets you Free: Literature, Liberty, and Western Culture.* Berman treats written language as a materialization of speech. While speech depends on face-to-face contact and on oral tradition, writing materializes speech by providing a visual representation of

[1] In reflecting on the relation of technologies to narratives I am inspired directly by the recent works of Mary Morgan on the way narrative functions in the use of models by economists (Morgan 2001, 2007). More generally, I have long followed the works of Hans-Jörg Rheinberger, for example, his thoughts on Historiality, Narration, and Reflection (1997, ch. 11): "Experimental systems contain remnants of older narratives as well as shreds and traces of narratives that have not yet been related" (p. 186). They are "generators of epistemic novelty" (p. 229). Other relevant sources, on narrative structure in evolution, primatology, and mathematics, are (Beer 1983; Haraway 1989; Alexander 2002), although they are primarily concerned with how structures of scientific understanding express broader cultural narratives.

oral expression. This materialization to some degree objectifies language (in the sense of making it into an object) and thereby provides a measure of autonomy to the written text. "The materialized realization of language in writing is the condition that allows for the autonomization of literature" (Berman 2007, p. 64).[2] Berman's argument is not one of technological determinism but of technological possibility, figured always within a social context of political, economic, and religious action. In complex societies and cultures, written language competes with other modes of symbolic expression (e.g., art) and different modes of writing compete with each other (literature, history, philosophy), but the process of symbolic abstraction is general. This process has reached its pinnacle in alphabetic languages—Greek being the canonical example—which suggests the place of "*Western Culture*" in Berman's title. I will add mathematical languages to this assessment.

The key contribution of written language, on Berman's account, is autonomy: autonomy of the text and with it autonomy of the writer and of the reader, in both space and time. In other words, writing may be said to provide our most pervasive "technology of distance" in space and time (Porter 1995, pp. ix, 14, 92, 200–208; 1999). Thus written language looks somewhat like Bruno Latour's "immutable mobiles" (Latour 1986, 1990, 1999). But like them, Berman's languages as technologies are actually neither immutable nor particularly mobile unless mediations of many kinds facilitate their movement, typically by continual transformation. Nevertheless, they do support traveling knowledge through objectification.

The autonomy of written language is also crucial to its role as a vehicle of critical reflection and creative imagination. This is as true in history and in science as it is in literature. Writing supports the capacity to create new ways of thinking and acting, as shown, for example, by Klein (2003). Such a claim would not be at all surprising if we were thinking only of the usual sort of material technologies: X-ray imaging, electron microscopy, PCR (polymerase chain reaction), GWAS (genome wide association studies), or fMRI (functional magnetic resonance imaging). We tend not to think of written languages in the same way, but we should. Their creative function reflects in part, I will argue, the capacity to support narratives of particular kinds about the objects of science. The narratives take on different forms in different areas and they change over time. It is this change in narrative form in mathematical physics that I will exemplify by contrasting explanations based on deductions from partial differential equations (PDE's) with explanations obtained from computer simulations.

3 Partial Differential Equations in the History of Physics

One of the most pervasive technologies in the history of physics, and certainly a technology of creative imagination, has been the continually evolving landscape of

[2] Although Berman wants to employ his views on language as a defense against postmodern historicist relativism, much of his discussion does not depend on that perspective. A key source, and foil, for Berman is (Ong 1982).

PDEs and methods of solving them. They constituted for nearly 200 years both the means and the goal of explanation. Canonical examples, roughly in chronological order, include: Lagrange's equations, Laplace's equation, the diffusion equation, the wave equation, the Navier–Stokes equation, Hamilton's equation, Maxwell's equations, the Schroedinger equation, and many others. The ubiquity of PDE's in physics is reflected in the standardized toolboxes of mathematical techniques that every physicist of the twentieth century learned to employ to deal with the array of systems and circumstances in which they could be applied. These texts-as-toolboxes came to be known simply by their authors' names, for example: Courant and Hilbert (1953) or Morse and Feshbach (1953). Courant stressed in his Preface to their English edition that they sought the unity of science through preservation of the relation between physical intuition and mathematics: "Mathematical Methods originating in problems of physics are developed and the attempt is made to shape results into unified mathematical theories" (Courant and Hilbert 1953, pp. v–vi; p. 265). Despite their appeal to physical intuition, however, Courant and Hilbert provide only the bare tools of mathematical methods. The physicist must come to the toolbox familiar already with a wide range of physical problems and an accompanying physical intuition, which the mathematical methods should help to develop and to refine. This intuition is essential to any creative use of the toolbox. Typically, it relates the physical problem to a PDE through a story about a piece of the world. It is this relation of story to mathematical structure that interests me here (Morgan 2001, 2007).

We can easily generate an instructive example for heat conduction, where we seek to explain how an initial concentration of heat spreads out through a conducting material. Modern physicists come to the problem knowing (at least implicitly) that Joseph Fourier analyzed it at length in his *Analytical Theory of Heat* of 1822 and that he treated it in terms of measureable macroscopic quantities—heat and temperature—and macroscopic parameters of the material—conductivity and heat capacity—independent of what heat might be.[3] Subsequently, we do not have to start out, as Fourier did, by reflecting on the nature of heat and by arguing against the then-reigning Laplacian view that physical explanations should reduce to microscopic interactions between atoms, although we do need to have a similar background story at hand.

Following the spirit of Fourier's analysis, we can begin our story of heat conduction with a picture, Fig. 1, of a uniform metal bar heated briefly at the center to produce an initial temperature distribution T_i and by imagining the bar to be infinitely long and perfectly insulated on all sides, so that it conducts heat only along its length. To appreciate the rendering of the story as a materialization, it is helpful to see it as actually being written out with pen and paper, associating the sketched diagram with a simplified mathematics, using Δ to symbolize a very small change. Within the idealized sketch we remark further that no heat should be lost in conduction, or that the heat leaving any element Δx of the bar $\Delta F/\Delta x$ should be

[3] For an account of the historical significance of this move from microscopic to macroscopic analysis for British physics, see (Smith and Wise 1989, pp. 149–168). It was critically important for such diverse areas as electrostatics (Green), the wave theory of light (MacCullagh), elastic solids (Stokes), and electromagnetic theory (Maxwell).

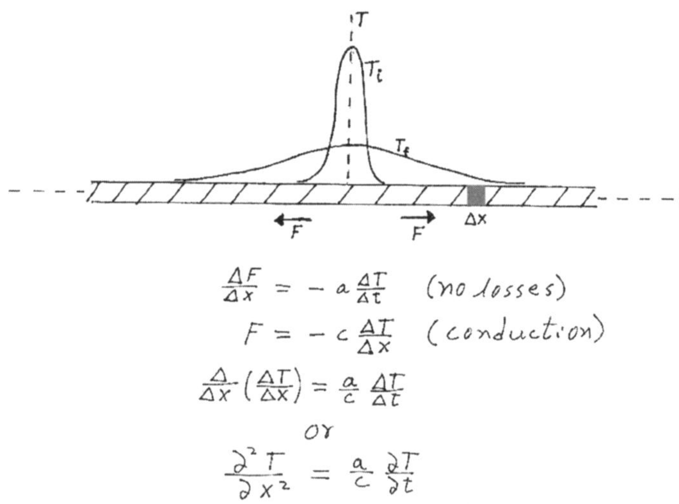

Fig. 1 Heat conduction in an infinitely long metal bar, with an initial temperature distribution T_i spreading in time to T_f

equal to its rate of decrease of temperature with time ΔT/Δt multiplied by the heat capacity *a*,

$$\Delta F/\Delta x = -a\Delta T/\Delta t.$$

Then we incorporate the empirically supported rule that the flux of heat at any point should be proportional to the temperature gradient there,

$$F = -c\Delta T/\Delta x,$$

where *c* is the conductivity of the bar. Substituting the second equation in the first we obtain,

$$\Delta(\Delta T/\Delta x)/\Delta x = (a/c)\Delta T/\Delta t.$$

This equation, however, is not mathematically rigorous. To put it into the formal language of partial differential equations, we need to call in our background story about atoms to justify treating a finite physical element of the bar, containing the many atoms that together contribute to the macroscopic parameters *a* and *c*, as an infinitesimal mathematical element having no microscopic structure at all. If the story is plausible, then we can carry out some standard mathematical arguments to convert Δ in the previous sequence into a partial differential ∂ by taking the limit as the element shrinks to zero.[4] The process will yield the partial differential equation,

$$\partial^2 T/\partial x^2 = (a/c)\,\partial T/\partial t.$$

This PDE, called the diffusion equation, should govern the development in space and time of any initial temperature distribution T_i into a later one T_f, as indicated in Fig. 1. There is nothing particularly striking about our narrative except that it

[4] Fourier's own rationale for this procedure was by no means standard. Poisson (1835) devoted an entire book to challenging its validity.

exemplifies a perfectly standard procedure for deriving PDEs in physics and that it is written out using a specialized language that physicists have long learned to speak and to write as the natural language of their subject. The result, as the written equation, gives a materialized and objectified representation of our thinking about the problem of heat conduction.

In order to interpret the consequences of our reasoning, we have now to find solutions to the equation. Luckily, physicists have only to open Courant and Hilbert to find a concise version of Fourier's famous solution, the Fourier series, along with a proof that the series will converge for any physically realistic (piecewise continuous) function.[5] They provide a deductive proof of the general form of the solutions. But deductions, to be physically meaningful—and this is now the basic point—require specific settings, where interpretation plays a crucial role.

Consider the analysis that William Thomson (later Lord Kelvin) presented in 1846. For his inaugural lecture as Professor of Natural Philosophy at Glasgow University, he entered the debate over the source of the earth's heat and the significance of the increase of temperature with depth observed in deep mines. Did the distribution result from a primitive central heat, gradually cooling over time, as Fourier had argued, or from other effects, such as the solar system having moved into a region of cooler temperature, as Poisson suggested? Thomson approached the problem by asking a question: could the diffusion equation always be run backwards from a present state to a preceding state. He distinguished three cases, depending on whether the Fourier series for a temperature distribution would converge or diverge. Divergence for any present distribution, he believed, implied that it could not be the product of any previous state, or could have no age; convergence for only a finite past time, preceded by divergence, would imply a finite age; while convergence for all past time would imply an unlimited age. Only adequate data on the present distribution of temperature would suffice for choosing between the alternatives, but Thomson's own beliefs certainly inclined him toward a finite age.[6]

His analysis depended on a subtle story about the distinction between physical impossibility and mathematical impossibility with respect to sharp temperature changes (cusps, corners, jumps, etc.), which he thought divergent. He would soon realize that this intuition was untenable. Nevertheless, it is apparent that although Thomson wrote out his argument in the form of a deductive proof, the diffusion equation carried with it a deeply meaningful creation narrative about the origin and age of the earth. This narrative would enter directly into his enunciation of the Second Law of Thermodynamics in 1850/51.

I stress the narrative aspect because we do not normally think of mathematical deductions as narratives. Indeed, Courant and Hilbert do their best to strip their mathematical methods of all narrative elements. And yet, the toolbox cannot be constructively employed in the world without putting the narrative back in. This is Mary Morgan's argument with respect to economic models.

[5] Courant and Hilbert (1953, pp. 69–73). For physical applications, they refer the reader to "elementary texts" (p. 4, n.1). The convergence proof for piecewise continuous functions was given by Dirichlet in 1829 (1889) and (1837).

[6] Discussion and references in (Smith and Wise 1989, pp. 192–194). Thomson may not yet have been familiar with Dirichlet's work on convergence.

> This activity of manipulating a model requires a narrative device, such as a question, which sets off a story told with the model. The structure or system portrayed in the model constrains and shapes the stories that can be told, but without stories showing how the structure works, we cannot tell what might happen in specific cases. Without these narrative elements, we cannot apply model-structures directly onto the facts of the economic world, nor demonstrate outcomes about the hypothetical world represented in the model (Morgan 2001, p. 361).[7]

To put this in the terms if my own example, the mathematical technology as language of partial differential equations and their solutions provides a (non-narrative) deductive structure to which physicists attach interpretive narratives about a wide variety of phenomena. That deductive structuring of the course of events has long defined what constituted an explanation in physics. The explanatory emphasis, however, has been on the deduction, to the exclusion of the attached narratives, and with that, the exclusion of anything like historicity in explanation.

4 Quantum Chaos

Turn now to more recent work in physics that begins to introduce historicity. PDE's are still the mainstay of analysis in many areas but their relation to explanation has changed fundamentally since the 1970s. Their weakness had always been that they were soluble only for relatively simple systems (e.g., the Schroedinger equation for the hydrogen atom with one electron) and became unmanageable for the vast majority of real-world problems (e.g., an atom with more than ten electrons), which typically involve intractable mathematics.[8] In these areas of complexity, computer simulations, whether beginning from a PDE or not, have become the means and perhaps even the goal of explanation. The temporal development of a robust simulation, typically followed visually on screen, now explains the dynamical behavior of the system. The technology of knowledge has changed dramatically. One cannot learn it from Courant and Hilbert. It requires instead mastering the languages of computational mathematics, non-linear dynamics, and associated visualization software. These new modes of speaking and writing have changed both what can be known and how it can be known. And they bring with them explanatory narratives that are more historical in form.

An example comes from the area of quantum chaos, itself a kind of oxymoron, since the quantum uncertainty principle ought to blur the sensitive dependence on initial conditions that characterizes classical chaos and should smooth out quantum

[7] In "Prisoner's Dilemma," Morgan (2007) illuminates how the apparently poverty-stricken 2 × 2 matrix for the famous dilemma acquires a rich and varied set of meanings through the stories with which economists surround and interpret it.

[8] Laughlin and Pines (2000) cite this situation and give a polemical critique of deductive and reductive quantum mechanics in their manifesto for the twenty-first century, arguing that a fundamentally different approach is required to deal with the complex systems of everyday life and their emergent properties. They call this new physics the study of "complex adaptive matter" (p. 30). They and their collaborators extend the analysis to mesoscopic systems, including bio-molecular systems, in (Laughlin et al. 2000).

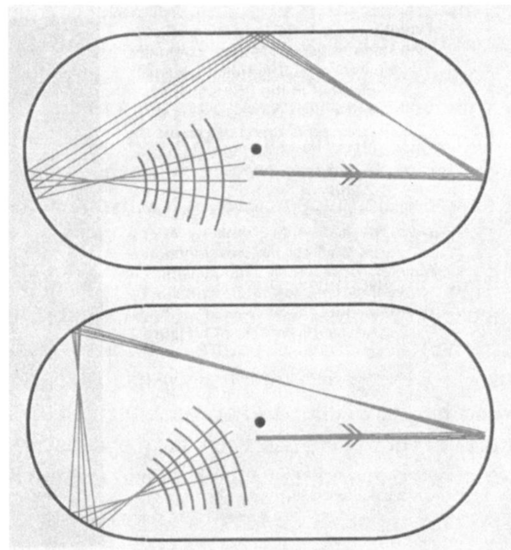

Fig. 2 Simulating the behavior of electrons in a stadium billiard (Heller and Tomsovic 1993, p. 43)

wave functions. A quite accessible discussion of representative phenomena was given some time ago by Eric Heller and Steven Tomsovic.[9] Consider a geometry called a stadium billiard (Fig. 2), for which the boundary may be thought of either theoretically as a wall that reflects electrons or experimentally as a ring of iron atoms implanted on a sheet of copper. Its size is important. It should lie in the mesoscopic border region between the microscopic world of quantum mechanics and the macroscopic world of classical mechanics (roughly, between ten and a thousand atoms). The problem is to find the wave function for electrons moving about in this billiard. The Schroedinger equation should govern the quantum wave function, but it is not soluble except by numerical means, defying physical insight, while the classical motion is chaotic, so that there are no stable classical orbits.

Nevertheless, using techniques that Eric Heller pioneered in 1984, he and Steven Tomsovic were able to find solutions by using a conceptually simple computer simulation (Heller 1984). It begins from a semi-classical depiction of bundles of particles bouncing around the enclosure. These substitute "electrons" are strange hybrid objects. They move like classical billiard balls but have a phase and amplitude associated with them. Therefore, they interfere constructively and destructively like waves, building up regions of higher and lower amplitude, to simulate quantum wave functions. "The picture is one of a swarm of trajectories carrying amplitude and phase around with them" (Heller and Tomsovic 1993,

[9] Heller and Tomsovic (1993). On the place of this work within trends in the physics of complexity see Wise and Brock (1998).

p. 42). That is, they are fictional objects which serve as tools for calculation, and yet, as the authors emphasize, they yield intuitive understanding and explanation of quantum phenomena. They do so, I will argue, as the subjects of a narrative describing their history, an evolutionary history generated by the simulation.

The simulation proceeds by generating many thousands of such swarms, beginning from the same point but with slightly different directions and velocities. After bouncing around the stadium a few times their summed amplitudes yield a wave function distributed through the entire space. Figure 3 shows characteristic wave functions (eigenstates) corresponding to what would be three different periodic orbits for classical particles, which are represented in the two lower pictures by heavy black lines (but omitting their symmetric twins). Along these periodic but unstable trajectories, where the particles would be reflected back and forth, the simulation reveals that the wave functions acquire high amplitude.

This remarkable result, which Heller named "scarring" in his original publication, is unexpected quantum mechanically. Yet the simulation makes it understandable in terms of the physical intuitions of classical mechanics. "Unavoidably we think classically about systems of more than a couple of particles." For Heller and Tomsovic the need for a classical narrative is a matter of our everyday experience rather than of any ontological priority. Their claims for classical understanding are pragmatic: "We will sidestep the philosophical debate and focus on the issues of useful approximations and physical insight" (Heller and Tomsovic 1993, p. 38). And yet, emphasizing the fact that new phenomena are revealed by the classical motion, they adopt causal language: "Periodic orbits that are not too unstable cause scarring" (Heller and Tomsovic 1993, p. 42). This attribution of cause to the classical trajectories might be thought of as simply a way of talking, indeed, as a way of forming a semi-classical narrative whose function is to show how the simulations assist intuition. But their role extends well beyond intuition. They also reveal physical properties that exist within the full quantum solution but lie outside of its frame of reference as "hidden dynamical symmetries" (Heller and Tomsovic 1993, p. 40). That is, they play an essential explanatory role not available otherwise. This is not to suggest that quantum theory is false, or even incomplete, but that in the mesoscopic domain both quantum and classical aspects are present simultaneously. Both theoretically and experimentally they form a realistic hybrid which requires both quantum waves and classical trajectories for explanation.

It is this strong explanatory role of the hybrid objects that I want first to emphasize. But the way in which their narrative connects to the mathematical simulation may not yet look so very different from the heat conduction example. I want also to stress, therefore, that the explanation of a scarred wave function is obtained only through the process of development in time, or "evolution," as Heller and Tomsovic express it.[10] The simulation generates an evolutionary explanatory narrative to replace the deductive narratives traditionally associated with PDE's like

[10] Biologists will insist that the process is not properly called evolution but only development. No doubt they are correct, but I will maintain the authors' broader usage here and below as an indication of their biological orientation.

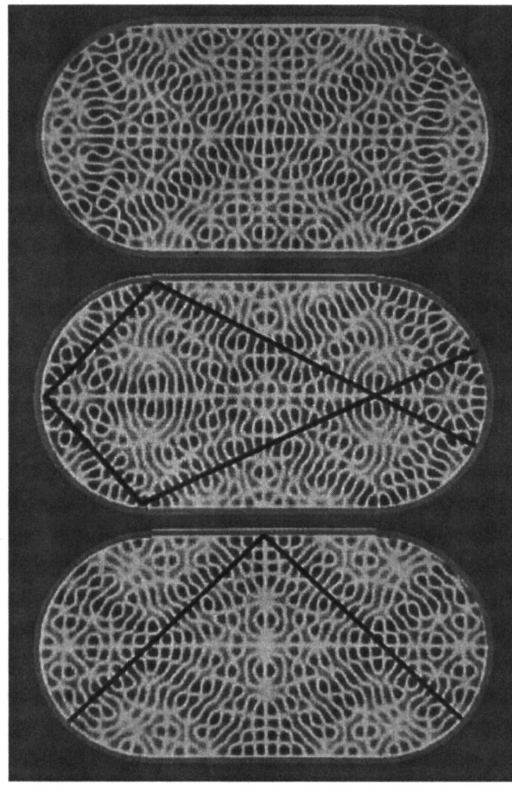

Fig. 3 Semi-classical simulations reveal "scarred" eigenstates, characterized by a buildup of amplitude along classically periodic orbits (*black lines*) (Heller and Tomsovic 1993, p. 40)

the diffusion equation and Schroedinger's equation, and this narrative aspect is now more explicitly present in the explanation. To evoke this aspect more directly, it may be helpful to put Heller and Tomsovic's evolving "swarm" in the form of a simple story, with reference to Fig. 4.

> Once upon a time there was a localized swarm of semi-classical electrons that set out on a series of journeys to explore a stadium billiard. It traveled initially toward the right hand end, while gradually diverging. Bouncing off that end, the spreading swarm propagated toward the left while being reflected also from the side walls, with interference between its manifold trajectories becoming increasingly complex. By the time its entire system of interfering trajectories had been reflected back and forth several times, it filled the whole stadium in a complex pattern of amplitudes, looking for all the world like a full quantum wave function. Upon repeating this exploratory process many

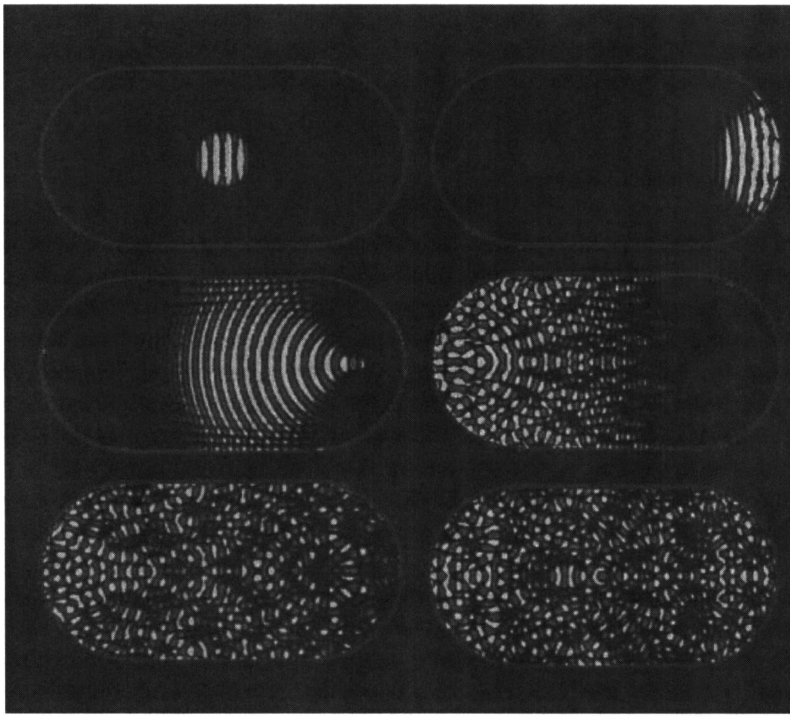

Fig. 4 Evolution of a wave function simulated semi-classically. A localized packet (*top left*), traveling initially to the *right*, is reflected back and forth until its interfering components fill the stadium (Heller and Tomsovic 1993, p. 44)

times, the packet discovered that it could generate a variety of special "scarred" patterns (Fig. 3), in which the scars revealed the effects of classical periodic trajectories that lay hidden within its dynamical structure. They persisted far longer than expected in the life of the wave function.

This narrative of an evolving swarm shares more with the narratives of natural history than with the deductive solutions of partial differential equations. At first glance it might seem that the difference is not so great, since PDEs can be said to control the evolution of a function in space and time, as we saw for the diffusion of heat. But as part of the deductive character of PDEs, proofs of the existence and uniqueness of solutions play a critical role. No such proofs exist for the simulated solutions. Their existence is discovered by the simulations, and the solutions are typically not unique. Instead, repeated many times, the simulations map out a space of possible solutions. They present a generative process that does not derive from any general law but is more like the growth of a seed into a plant. And it is the morphological properties of the "plant" that develop during the period of growth—here the symmetric orbits and their periodicities—that occupy the center of attention. They are objectified and materialized, both in the mathematics and on screen, through the languages in which their evolution is written. In this sense, the distributions that emerge (the explananda) are essentially historical objects; furthermore, they can be understood only through their histories (explanans). I will return to this agreement of object and explanation in conclusion.

5 Physics as Natural History

Natural historians have always been interested in morphologies, their diversity, and their patterns of growth, but only relatively recently have physicists begun to employ the natural-historical mode for exploring the architectures of nature. An example is the study of snowflakes. Perhaps most of us will think of the typical snowflake as exhibiting an intricately beautiful geometrical pattern, highly symmetric, with six identical arms. This is the image whose explanation the witty Kepler reduced to "nothing" in his *Six-Cornered Snowflake*. Descartes too (Fig. 5a) highlights the hexagonal symmetry, noting that it is "impossible for men to make anything so exact." And Robert Hooke (Fig. 5b) ascribes this perfection of snowflakes to the "Geometrical *Mechanisme* of Nature" (Kepler 1966; Descartes 1965, pp. 313f; Hooke 1961, p. 91).

This familiar assumption of ideal mathematical form as the foundation for order and beauty in nature continued to govern studies of snowflakes from the 17th through much of the twentieth century; despite widespread recognition that full hexagonal symmetry was very rare. In a move that is still typical, Hooke ascribed the irregularity that he regularly observed under the microscope to "the thawing and breaking of the flake by the fall, and not at all to the defect of the *plastick* virtue of Nature." And still in 1931 (Fig. 6) in a classic collection of photomicrographs by W. A. Bentley, known for their exquisite beauty, only two of nearly 200 plates contain nonsymmetric images, with the remark from his physicist (meteorologist) collaborator that they "may be attributed to fortuitous disturbances of one kind or another in the course of their growth" (Hooke 1961, p. 91; Bentley and Humphreys 1931, p. 14).

There is at least one major exception to this rule, contained in a big volume by Ukichiro Nakaya, published in English in 1954, summarizing his extensive work

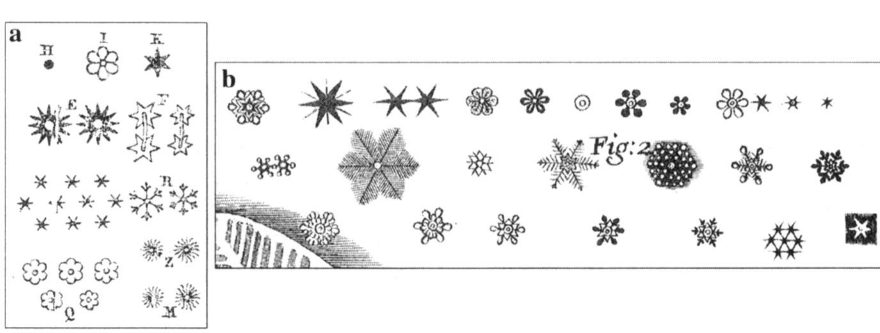

Fig. 5 Sketches of snowflakes by **a** Descartes (1965, 312) and **b** Hooke (1961, facing 88)

from the 1930s and early 40 s (Nakaya 1954).[11] Trained as a nuclear physicist, Nakaya found himself with a position at Hokkaido University in the north of Japan, with no facilities for nuclear research but with plenty of snow. So he turned with his students and collaborators to taking photomicrographs of snowflakes, both the natural sort and those they could produce artificially while spending long hours in a very cold laboratory (Fig. 7a). It contained a chamber for growing snowflakes under variable conditions of temperature and humidity. Figure 7b shows his first artificial flake. Finding that "a perfectly symmetric snowflake is very rarely observed," whether on the slopes of Mt. Tokachi or in the laboratory, Nakaya studied non-hexagonal symmetry and irregularity, which immediately focused attention on the normal processes of growth, rather than only on supposed states of perfection. Growth, it seems, is not easily subsumed under ideal geometrical forms. Figure 8a, b, c reproduce a few of his photomicrographs. Figure 8c, showing stages of growth, merits comparison with traditional sequences from natural history, such as the seventeenth century images in Fig. 9, showing the life history of a frog from egg to tadpole to maturity and the development of a seed into a flowering plant (Swammerdamm 1682, plate XII).

Such a zoo of irregularity, asymmetry, and growth, even though produced under controlled conditions in a laboratory, was not the sort of thing that interested most physicists in the 1950s, particularly not elementary particle physicists, for whom elementary particles were ahistorical mathematical objects, eternal and universal. They sought explanations within elegant mathematical models. Nakaya's snowflakes gained little recognition.

The climate for work like Nakaya's changed dramatically during the 70, 80, and 90 s as problems of complexity became ever more important in mainstream physics research.[12] It is only very recently, however, that a physicist at California Institute of Technology has taken up snowflakes as part of his work on pattern formation in nonlinear, nonequilibrium systems. Kenneth Libbrecht has extended Nakaya's natural and artificial crystals with much higher resolution equipment (Fig. 10). In 2006 he published what he calls a *Field Guide to Snowflakes*. Libbrecht's *Field Guide* and his extensive website contain photomicrographs of an amazing diversity of natural forms (Libbrecht 2006, 2011a). I take the term "field guide" to be explicit recognition that natural history and the study of nonlinear dynamical systems have much in common, especially complexity. Indeed, Libbrecht writes about snowflakes in terms of their "life history," constructing a "story" much like the one I suggested above for quantum chaos. The story yields a diagram and a lesson: "Complex history [yields] Complex crystal shape" (Libbrecht 2011b).

That lesson is apparent also in the work of two of Libbrecht's collaborators, who do simulations. Even a decade ago it was not practicable to simulate the evolution of a snowflake at high resolution. But the mathematicians Janko Gravner at University of California Davis and David Griffeath at University of Wisconsin Madison have

[11] Lorraine Daston has recently made me aware of an earlier exception (Hellmann et al. 1893). See Daston et al. (2007, pp. 148–155).

[12] Even the Big Bang theory and evolution of the cosmos appear not to have turned elementary particles into historical objects, presumably because they are not known by evolutionary means.

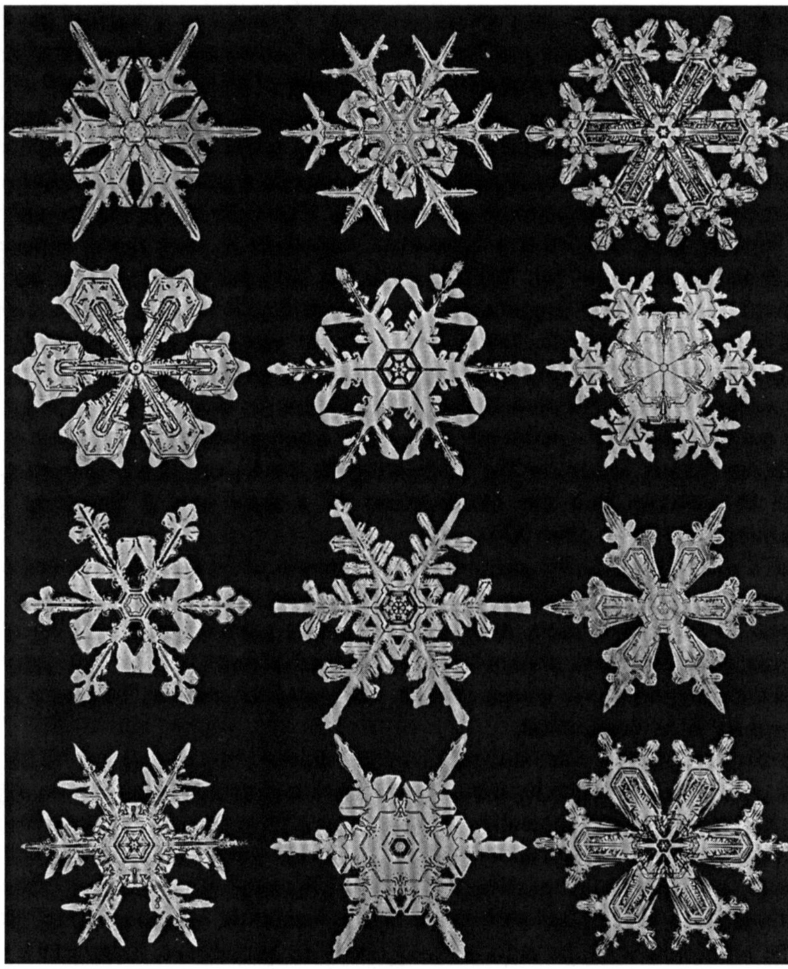

Fig. 6 Photomicrographs of snowflakes (Bentley and Humphreys 1931, p. 147)

produced a three-dimensional, mesoscopic, computational model that replicates many of the basic forms or "habits" of snowflakes—dendrites, needles, prisms, etc.—along with their more intricate "traits"—sidebranches, sandwich plates, hollow columns, and a variety of surface effects, or hieroglyphs: ridges, flumes, ribs, circular markings, and other more chaotic patterns (Gravner and Griffeath 2009, traits, p. 1; habits, p. 17). Figure 11 shows several examples.

Gravner and Griffeath employ a conceptually simple computational model, which grows a virtual snowflake from a small seed of ice surrounded by water vapor and governed by only three mechanisms: diffusion of water vapor from the crystal; freezing and melting in a narrow boundary layer; and attachment rates at the boundary that favor concavities. Despite this conceptual simplicity, however, implementation of the model in a continually updating cellular automaton requires many parameters and therefore large amounts of computing time. For example,

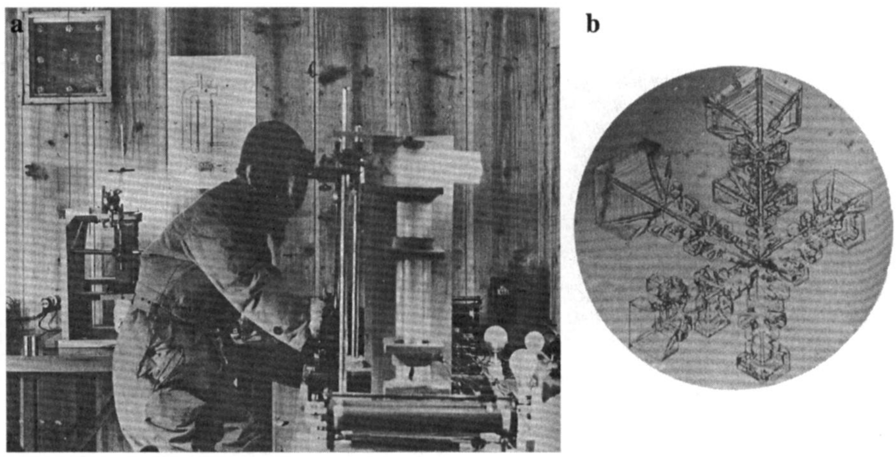

Fig. 7 **a** Laboratory with chamber for growing snowflakes and **b** first artificial snowflake (Nakaya 1954, pp. 144, 152)

because the units composing the virtual crystal are taken to be hexagonal prisms, attachment rates for new units have to be specified for those that attach in all possible combinations of ends and sides (Fig. 12). A similar multiplicity applies to melting and freezing rates. Density and drift velocity add two more parameters. Although the total number could be reduced by making realistic assumptions, free parameters in the model remain numerous, even when considering only fully symmetric arms (twelve identical half-arms) and top–bottom symmetry. The evolution of a single such snowflake takes about 24 h on a modern desktop computer.

Gravner and Griffeath forthrightly acknowledge that it is not very clear just how their intuitively plausible parameters correlate with physical processes and that their simulations do not treat important issues of non-symmetry, randomness, singularities, and instabilities, that is, the full panoply of contingencies that affect growth. They nevertheless believe that the evolutionary simulations provide "explanations" of many of the characteristics of natural snowflakes, both in general morphology and in the details of their traits. Run many times over, with varying parameters, the simulations explore the space of possible snowflakes and their probable mechanisms of formation, somewhat as we have seen for quantum chaos. These explorations too discover new properties, such as a "sandwich instability" (which accounts for the fact that most snowflakes actually consist of two-plate sandwiches) and they suggest new phenomena that will require new kinds of laboratory observations on real snowflakes, such as ridges growing in the interior between two plates rather than on the outside surfaces.

Even more than in the case of quantum chaos, the explanations and discoveries obtained in the photomicrographs and simulations of snowflakes are natural historical in kind. Key terms are *trait*, *habit*, *morphology*, *seed*, *life history*, *evolution*, *field guide*. The simulations not only deal literally with objects of natural history, generating a kind of botanical garden of snowflakes, but the traits of the different genera and species in the garden are explained by the conditions of their development, read as evolution. The principles governing the evolution may not

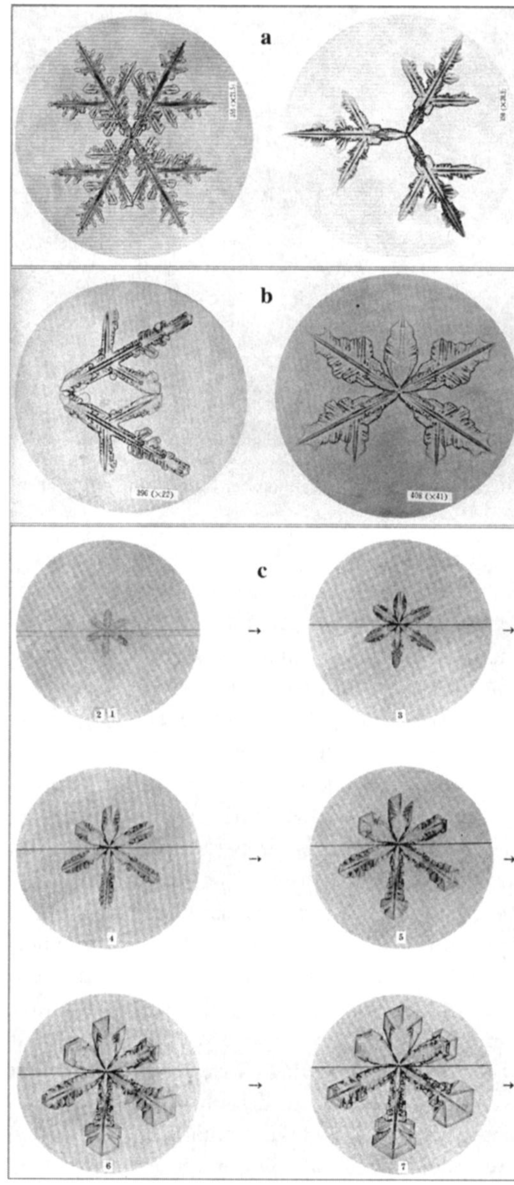

Fig. 8 **a** Three and four-fold symmetry, **b** irregularities, and **c** growth (Nakaya 1954, pp. 383, 389, 257)

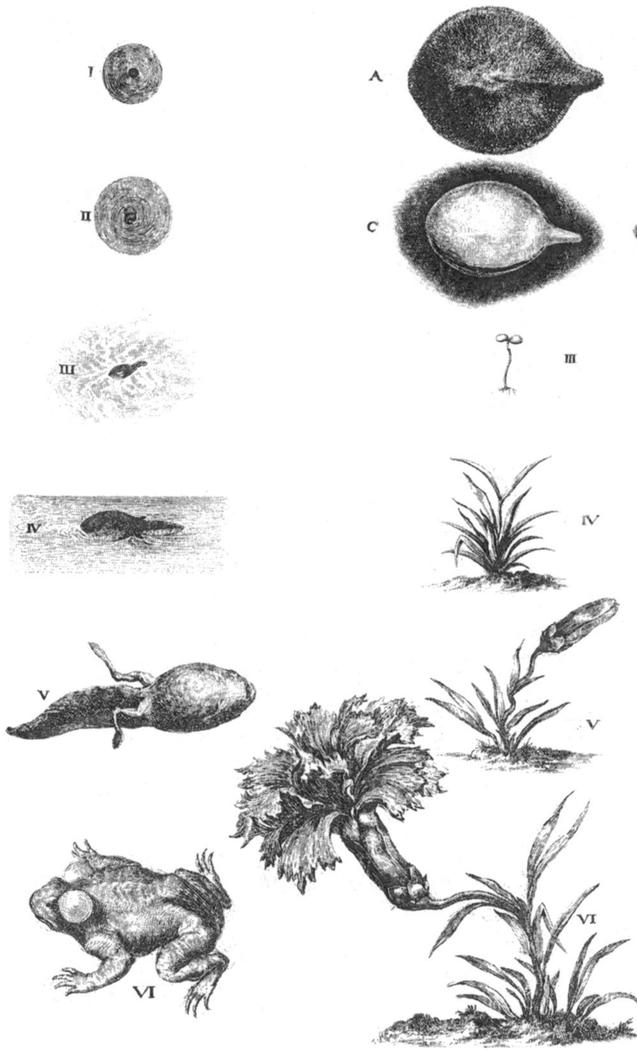

Fig. 9 Stages of growth of a frog and a plant (Swammerdamm 1682, plate XII)

be the Darwinian principles of variation and selection but they are nevertheless simple generative principles capable of explaining how the entire phylogeny derives from something like a common ancestor developing under varying environmental conditions.[13] That is, the simulations generate an evolutionary narrative that explains

[13] A better analogy might be the growth of many different kinds of tissues from a single kind of stem cell. Laughlin et al. (2000) also invoke "evolution" (p. 35)—along with growth (p. 32), aging (pp. 32, 34), and adaptation (as in complex adaptive matter, or behavior, p. 36)—to capture the analogy of physical to biological processes. Indeed they offer the hint that the phenomenon of protection (independence of a mesoscopic physical system from small changes in microscopic structure or laws) may in biology "arise from the necessity of tolerating diversity."

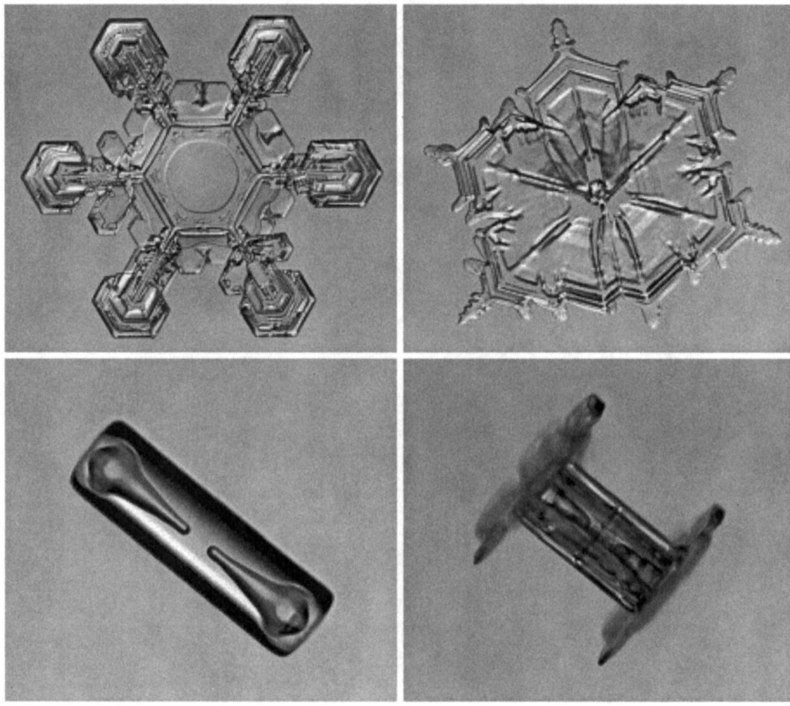

Fig. 10 Photomicrographs of snowflakes (Libbrecht 2011a)

the natural order of snowflakes as an essentially historical order. Every individual is a unique product of its history, full of contingencies and accidents. A snowflake is an historical object and its virtual history is its explanation. This is a long way from the traditional explanatory narratives of physics, which treat physical objects as timeless mathematical things explained by deductive solutions of partial differential equations. And of course the language of growth replaces that of deduction and reduction.[14]

Finally, the role of visualization requires comment. Visual images have always been crucial in physics to guide intuition and reasoning and to illustrate problems and solutions. But the role of visualization, and of languages of visualization, in the explanatory narratives generated by simulations is qualitatively different, for it typically serves as the direct representation, and the only effective means for understanding, the growth process and its intricate results. The simulation, to be understandable, must incorporate a technology for converting the calculations performed by the cellular automaton into an object accessible to the senses. Thus

[14] Explanations of this kind are not so surprising for simulations in field sciences like geology, but the point is the same: simulations often explain by supporting natural histories and their narratives (Oreskes 2007).

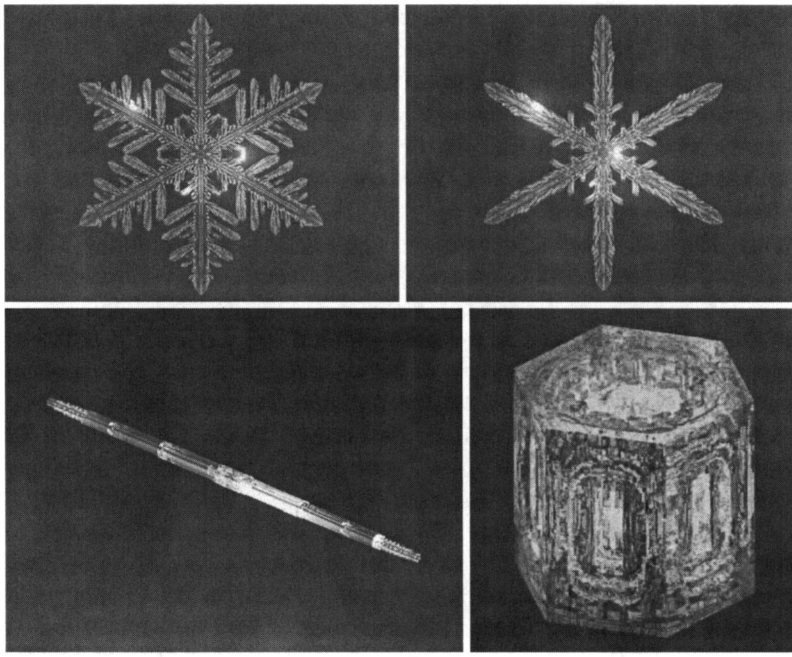

Fig. 11 Simulated snowflakes (Gravner and Griffeath 2009, p. 13)

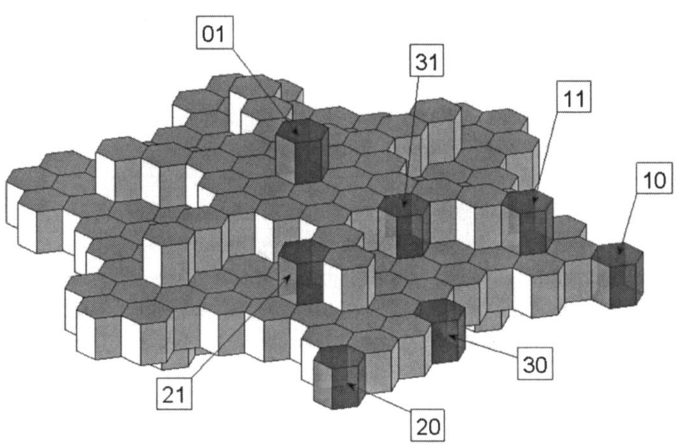

Fig. 12 Basis for simulating the evolution of a snowflake by attachment of hexagonal prisms: 0–3 sides and 0 or 1 ends (Gravner and Griffeath 2009, p. 3)

Gravner and Griffeath employ MATLAB and POV-RAY software to make their cellular automaton visually legible, with a resolution suitable for direct comparison with photomicrographs. Slide shows and movies accompany these new technologies.

6 Etruscan-Tuscan Simulation: Evolution of the Population of Tuscany

So far I have wanted to show how historicity, in the sense of natural history, has entered explanations in physics through new technologies/languages of simulation that generate evolutionary narratives. But lest these narratives do not yet seem properly historical, I take up a more obviously historical simulation and use it to reflect back on history itself.

Consider the long-standing debate over the relation of present-day Tuscans to their supposed ancestors, the Etruscans. Many Tuscan families steadfastly believe that they derive from the Etruscans, while archaeologists and linguists contend that the Etruscans disappeared as a distinct cultural and linguistic population after they were granted Roman citizenship in the second century B.C. In an attempt to generate an evidentiary account of the Etruscan-Tuscan relation, a group of anthropological geneticists, or "genetic historians," at the University of Ferrara and Stanford University have used genealogical simulation techniques to investigate a wide variety of "historical scenarios" (Belle et al. 2006). They began with ancient mitochondrial DNA from the bones in Etruscan tombs, obtaining viable samples from 27 individuals characterized by 22 haplotypes, and they compared these Etruscan sequences with modern mtDNA from people in Tuscan towns located in the former Etruscan area of Etruria, with 49 individuals characterized by 40 haplotypes. A software program called SERIAL SIMCOAL (for serial coalescent simulation proceeding backwards in time) produced candidate genealogies connecting the two samples, representing potential models of evolutionary relationships between Etruscans and Tuscans. The simulations ran over 100 generations (2,500 years) using different models—I would add narratives—of demographic events and social structure, with 1,000 simulations conducted for each model. Judgments of goodness-of-fit of each model in comparison with the empirical data were then based on median values from each set of 1,000 replicates for several measures of diversity within and between populations.

The models (and narratives) for various historical scenarios (Fig. 13) were basically of two types: single population models and two population models. The single population models (top), representing direct descent from Etruscans to Tuscans, included several variants: population of constant size, exponentially expanding population, expanding population with preceding founder effects and alternative mutation rates, and an original expansion followed by a severe bottleneck (Model 5), representing poor Etruscan living conditions after assimilation into the Roman state (i.e., negative selection). None of these models of genealogical continuity matched very well the observed data on haplotypes for Etruscans and Tuscans. The models for two separate populations of Etruscans and Tuscans (bottom) began with an initial divergence 240–500 generations ago, preceding expansion in either population. Then various scenarios of expansion and migration were added, with either mutual migrations from 240 to 100 generations ago or more recent migrations of Etruscans to Tuscans. Finally, the possibility was considered of elite dominance by a small subset of migrating Etruscans from abroad whose DNA just happens to be what was preserved in the

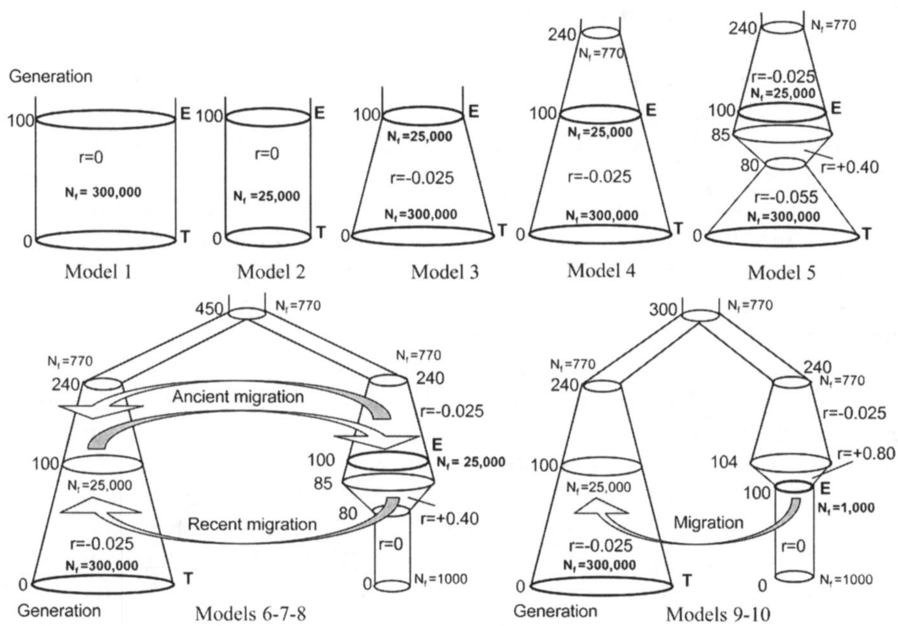

Fig. 13 Evolutionary models relating Etruscans (E) to Tuscans (T) (Belle et al. 2006, p. 8016)

burial sites. They might have formed a social elite who dominated the local Tuscan population and perhaps mixed with them. Of these alternatives, the ancient migration model gave the best fit to the observed data, indicating that the two populations (Etruscans and Tuscans) became independent over 100 generations ago, so that any genealogical relationship since then is very weak, or so it seemed initially.

But the historical scenarios did not quite end there. In more recent work (unpublished) these genetic historians have considered modern Tuscans from three different areas separately—Casentino, Murlo, Volterra—and have used the simulation procedure to compare them individually with the Etruscans. The data from Murlo and Volterra once again fit a two population model, in which the Etruscans may continue into the medieval period but then disappear entirely. But to everyone's surprise, data from the Casentino valley, long known for its isolation within Italy, does seem to fit a single population model (analogous to Model 5 of Fig. 13) beginning from the Etruscans and continuing to expand through the medieval period. The population undergoes a sharp contraction corresponding with the Black Death of the mid-fourteenth century, and then expands again up to the present. This remarkable finding of genealogical continuity depends entirely on subdividing modern Tuscans into very local populations, more isolated than most historians would have thought credible, while supporting the Casentinian claim to being direct descendents of the Etruscans. They can now tell a more elaborate and credible historical narrative to explain their origins.

7 Conclusion: We Know What We can Grow

In the preceding examples I have moved from an ahistorical deductive account of heat conduction through three increasingly historicized narratives about electrons, snowflakes, and Etruscans, attempting to draw out how their historicity is produced by the technology/language used to realize them, that is, to grow them, and thereby to know them. But if mathematical physicists have begun to act a bit like historians, what is it they have learned, and what can we learn from their efforts?

Each simulation begins with a so-called "scenario" that gets the history started and proceeds through an evolutionary development to a possible later state. The plausibility of the associated story depends on the continuity provided by the evolving series of states and on the degree to which this evolution can incorporate the full range of empirical information available. So far this looks pretty much like a form of what historians normally do in exploring historical narratives. Historians are constitutionally empiricist. They generally attempt to write a story with a beginning, middle, and end which incorporates as much factual information as possible into a coherent account, where coherence depends above all on continuity of people and processes. The history may be written from a cultural, social, political, economic, or other perspective; the facts may derive from a wide variety of sources; and interpretation and judgment are always in play, but in general, conviction comes with narrative coherence and empirical adequacy. This much will seem unproblematic to most historians. But the analogue in simulated histories may help to remind us of what historicity means at a deeper level.

There exists a large philosophical and historical literature on explanation in narrative history, revolving around a debate over whether narratives can "explain" at all and if so how. The canonical reference point for this debate, extending from the 1960s to the present, is Carl Hempel's articles of 1942 and 1963 on explanation in history (Hempel 1965a, 2001). Hempel's view that explanation requires subsumption under general laws, or covering laws, reflected his implicit assumption that physics supplies the model for all natural science and that explanations in physics rest on the sort of deductions from PDE's with which I began (or on statistical laws, which I omit here).[15]

When the analytic philosopher of history Arthur Danto took up the defense of narrative explanation in the 1960s, he argued that one could inscribe historical micro-changes ("atomic sentences") into the Hempelian mold of general laws governing causal connections between events, while nevertheless insisting that for the macro-changes ("molecular sentences") of narratives describing long-term developments "no general law need be found to cover the *entire change*" (Danto 1985, p. 255). His virtuoso logic no doubt establishes that narratives can explain if

[15] Hempel's brief discussions of "historic-genetic" or simply "genetic" explanation (Hempel 1965b, pp. 447–453; Hempel 2001, pp. 287–289), by which he referred to an account of an event by tracing its origins, or genesis, might seem promising for narratives and simulations, but Hempel quickly reduced any valid genetic explanation to a sequence of stages connected by nomological explanation, perhaps combined with description.

deductions from general laws can, but it does little to illuminate how narratives are related to explanations in natural science that do not depend on general laws.

More literarily inclined philosophers of history, like Hayden White (1973) and Paul Ricoeur (1984), have argued that narrative history is more like fiction than natural science. Simplifying their deep and subtle explorations, the implication seems to be that narrative explanation either cannot be judged in terms of whether it establishes objective truths about events in the real world (White) or at least that the emphasis needs to be placed on narrative's creative role in transforming such events into something quite different (Ricoeur). Yet a third strand, represented by David Carr (1986), rejecting both the relevance of the general law approach to historical narrative and the disconnect between such narratives and the real world, argues that narratives are deeply grounded in our everyday human experience of the world in time and that they explain by providing stories that match that temporal experience.

Perhaps the most striking feature of this ongoing debate is that it so consistently over the years has taken the Hempelian model of scientific explanation as its reference for comparing narrative with science.[16] That assumption runs throughout a 2008 forum in the journal *History and Theory* on Historical Explanation. David Carr, in the lead article, surveys the debate and his own position in it, regularly associating the "scientific" with Hempel's covering law model and the claim that "any genuine explanation must be in keeping with a causal-scientific approach borrowed from physical science. Today, of course, it is biological reality that serves in this role. As we've seen, the reduction of all reality to physical reality goes hand in hand with a reduction of all science to physical science as the preferred model of scientific explanation." (Carr 2008, p. 28). The succeeding three articles in the forum, while pursuing other interests, either reinforce this image of science or do nothing to contest it.

The problem with this long tradition in philosophy of history, of course, is that it is so completely out of touch with developments in natural science since the 1970 s. Hempelian explanation, as I have indicated by contrasting heat conduction with quantum chaos and snowflakes, has little purchase in the sciences of complexity, and there are no covering laws in biology, at least none from which anything very specific could be deduced. The philosophy of history, therefore, as currently practiced, offers little ground for exploring the concept of narrative explanation within the natural sciences, where it has been supposed not to exist.[17] Nevertheless,

[16] This despite Danto's explicit belief that he represented the historicizing "revolution" initiated by Thomas Kuhn and Norman Hanson against the Hempelian view of science (Danto 1985, p. xi). The revolution, however, did not stress science as narrative, nor did it attack deduction as explanation. It did insist that observation is always theory-dominated and subject to interpretation, which can be seen as introducing a narrative thrust, especially in history of science.

[17] See, however, Ricoeur's brief reference to explanations in "cosmology, geology, and biology" and certain parts of history as explanations in which retrodiction is at work to establish necessary conditions for something to have happened but prediction based on sufficient conditions is not possible (Ricoeur 1984, 135).

the efforts of philosophers to articulate the properties of narrative history and how it explains raise several points of interest here.

7.1 Object and Explanation

Carr's basic point is that "narrative is at the root of human reality" and "It is because of [the] closeness of structure between human action and narrative that we can genuinely be said to explain an action by telling a story about it" (Carr 2008, p. 29). His focus is on the intentionality of conscious human agents. If such intentionality were necessary for historical narrative, it would have no analogue in natural science. Much historical narrative, however, is written without reference to intentions, especially social and economic history,[18] but also histories of prehistoric humans, non-human animals, and every sort of living or non-living thing. In any case, Carr's general argument seems much broader than intentionality: "the point of emphasizing the sameness of form between narrative explanation and what it explains is to show that the narrative explanation does not inhabit a different conceptual universe from the explained" (Carr 2008, p. 29). He may be saying something like this: the validity of an explanation depends on the explanation mirroring what we take the properties of the object of explanation to be. On this reading, the successful deduction of simple quantum wave functions from the Schroedinger equation explains them because we believe them to be objects whose development is governed by that equation, a belief justified by agreement with observations. But when the deductions fail, as they do for the mesoscopic domain, another form of explanation is required, which Heller and Tomsovic obtain by growing the distribution in a simulation, incorporating a semi-classical narrative taken to represent the complex entanglement of quantum and classical properties. The growth narrative plays a critical role. It is for this reason that I have referred to the quantum distributions and to snowflakes as historical objects. The best way to explain their properties (perhaps the only way) is to write their histories, using sophisticated new technologies of writing. The same is true of the Casentinians, who more closely fit Carr's criterion of human agency, even though we know nothing of their intentionality.

Presented in this way, an explanation via a simulation with an associated growth narrative might be thought of as a general form of explanation of things that develop in time, of which a deduction from a PDE is a subset. In a limited domain, or in relatively simple situations, the deduction may supply the need for explanation, namely, where it provides a direct solution that develops in space and time, as from T_i to T_f in the heat conduction example. A simulation could be done to find the solution even in that situation, although deduction may be more efficient. In short, we need not think of simulations as an inferior stand-in for a preferred and ideal form of explanation by deduction; they might be said instead to reach for a more satisfying form of explanation, via historical narrative.

[18] I leave out of account the belief of the *Annales* school that social history, because quantitative and employing mathematical models, is scientific rather than narrative, which is precisely the dichotomy I am rejecting.

7.2 Chronicle and Narrative

A second issue, and one that goes back to Arthur Danto, is that a historical narrative is much more than a chronicle of events and in fact could not be written by a participant in the events (Danto 1985, ch. 8). The historian must be able to pick out what will be relevant in relation to future events, which can only be known in retrospect. This reflection leads to Danto's concept of "narrative sentences," which move a story from earlier to later moments and which could not be written by a mere chronicler. One might ask whether simulated histories are anything more than chronicles, and thus not narratives at all. But I believe that, in this respect, simulators are in much the same position as historians. They must run possible simulated histories in order to arrive at known or knowable outcomes, much like historians exploring various historical narratives in order to arrive at the most plausible ones.

7.3 Causality and Contingency

A somewhat different question related to the chronicle problem is whether simulations generate a mere sequence of states, with no identification of what connects them, such as a law, perhaps causal. I think this is not the case. In the continuously updating cellular automaton for snowflakes, for example, the possibilities for each successive state depend very sensitively on the preceding state, on the vicissitudes of the environment, and on instabilities. With respect to the growth of any particular snowflake, the succession of states is similar to that for all complex human processes, in which the contingencies are so great that lawlike histories of why particular kinds of events happen are not likely to be credible. More forcefully, if physicists cannot give a lawlike account of a snowflake, why would anyone seek to give a lawlike account of the French Revolution or of why Darwin took up eugenics. What historians can do well in their narratives, however, is determine in retrospect the conditions under which people had the resources and motivations to do what they in fact did do. These are better described as conditions of possibility rather than as determining conditions or causes that generate an outcome. Similarly, to run a simulation backward, as in the Etruscan case, is to find the conditions of possibility (or not) for Tuscan descendents of Etruscans, perhaps the Casentinians.

Historians are quite conscious of how contingencies constantly disrupt lawlike or causal stories. On the other hand, the meaning of contingency itself is a vexed issue. Philosophers like John Beatty have written penetrating papers on the subject for evolutionary biology, and it would be good to extend their nuanced reflections to contingency in history (Beatty 1995, 2002, 2006). I note here only that for most historians such contingency should at least include the contingency of outcomes, meaning that events could have turned out very differently under slightly different conditions.

7.4 Local and Global

Because contingencies matter, so does locality and specificity. This is another common theme of historical narratives that reappears in simulations. The differences in the particular paths of snowflakes falling for an hour through changing conditions of temperature and humidity produce very different formations. Similarly, Casentino and Volterra grow rather different Tuscans, just as mid-nineteenth century Berlin and Paris grew recognizably different physicists. On the other hand, nothing is fully local. Local meanings and dynamics depend on more global developments. In this the simulations of electrons in a stadium and even of snowflakes in the atmosphere are perhaps too self-contained to be properly historical. They cannot respond to changing conditions outside their self-reflexive algorithms (although nothing in principle would prohibit more extended environmental considerations). The Etruscan simulations are more historically realistic in this respect because their scenarios can accommodate a considerable range of historical developments. Casentinian history, for example, depends on Roman conquest and the Black Death. But historians too are quite limited in their capacity to interrelate the detailed explanatory power of local narratives with more global concerns. Satisfaction no doubt lies in moving back and forth between different levels of explanation.

These various aspects of the analogue between simulations and historical narratives all point to a very general lesson: explanations of the behavior of complex systems may always require a turn to historical narratives. Natural scientists seem to be taking on that view (see Libbrecht's lesson, above), albeit less in association with history itself than with natural history and biology. In recognition of the new order, historians and philosophers of science need a new epistemological slogan to replace some of the baggage left over from studies of the Scientific Revolution and the Enlightenment, which made abstractions from working machines, in the form of Newton's Laws and Lagrange's Equations, the cradle for all scientific explanation. To put it differently, we have lived too long with the adage of the mechanical philosophy: "We know what we can make," or in the version of the historian Vico, "the norm of the truth is to have made it" (Costelloe 2003). An appropriate update for the era of simulations would be: "We know what we can grow." Or concretely in terms of the Etruscan simulation: Etruscan DNA lives on in Casentino, *probably*.

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